Introduction to Statistical Quality Control

6th Hour

Based on the book “Introduction to Statistical Quality Control”, 7th Edition by Douglas C. Montgomery.

STATISTICAL QUALITY CONTROL

Statistical Basis of the Control Chart

Chance and Assignable Causes of Variation

- A process is operating with only **chance causes of variation** present is said to be **in statistical control**.
- A process that is operating in the presence of **assignable causes** is said to be **out of control**.

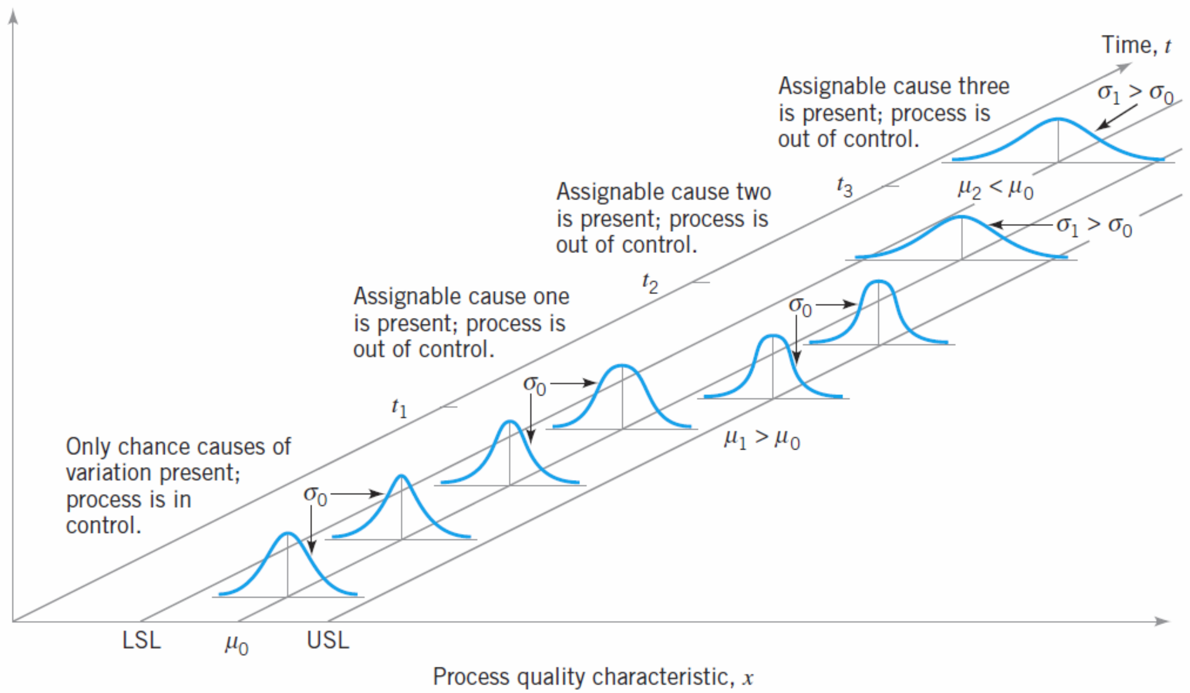


FIGURE 5.1 Chance and assignable causes of variation.

Statistical Basis of the Control Chart

- A control chart contains
  - A **center line**
  - An **upper control limit**
  - A **lower control limit**
- A point that plots within the control limits indicates the process is in control
  - No action is necessary
- A point that plots outside the control limits is evidence that the process is out of control
  - Investigation and corrective action are required to find and eliminate assignable cause(s)
- There is a close connection between control charts and **hypothesis testing**

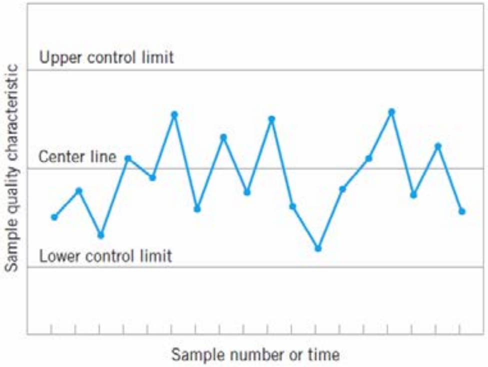


FIGURE 5.2 A typical control chart.

Shewhart Control Chart Model

- If we know that $X \sim N(\mu, \sigma^2)$ then

$$\bar{X} \sim \mathrm{N}(\mu, \frac{\sigma^2}{n})$$

$$\frac{\bar{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}} \sim \mathrm{N}(0,1)$$

- Thus,

$$\Pr(-3 < \frac{\bar{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}} < 3) = 99{,}73\%$$

Shewhart Control Chart Model

- Moreover,

$$\Pr(\mu - 3\sqrt{\frac{\sigma^2}{n}} < \bar{X} < \mu + 3\sqrt{\frac{\sigma^2}{n}}) = 99{,}73\%$$

- If we set $\sigma_{\bar{X}} = \sqrt{\frac{\sigma^2}{n}}$

- for each sample we should have

- $\mu + 3\,\sigma_{\bar{X}} < \bar{X} < \mu + 3\,\sigma_{\bar{X}}$ with probability 99,73% or

- $\bar{X} > \mu + 3\,\sigma_{\bar{X}}$ with probability 0,135%
- $\bar{X} < \mu - 3\,\sigma_{\bar{X}}$ with probability 0,135%

Shewhart Control Chart Model

We may give a general **model** for a control chart. Let w be a sample statistic that measures some quality characteristic of interest, and suppose that the mean of w is μ_w and the standard deviation of w is σ_w. Then the center line, the upper control limit, and the lower control limit become

$$\begin{aligned} \text{UCL} &= \mu_w + L\sigma_w \\ \text{Center line} &= \mu_w \\ \text{LCL} &= \mu_w - L\sigma_w \end{aligned} \qquad (5.1)$$

where L is the "distance" of the control limits from the center line, expressed in standard deviation units. This general theory of control charts was first proposed by Walter A. Shewhart, and control charts developed according to these principles are often called **Shewhart control charts.**

How the Shewhart Control Chart Works

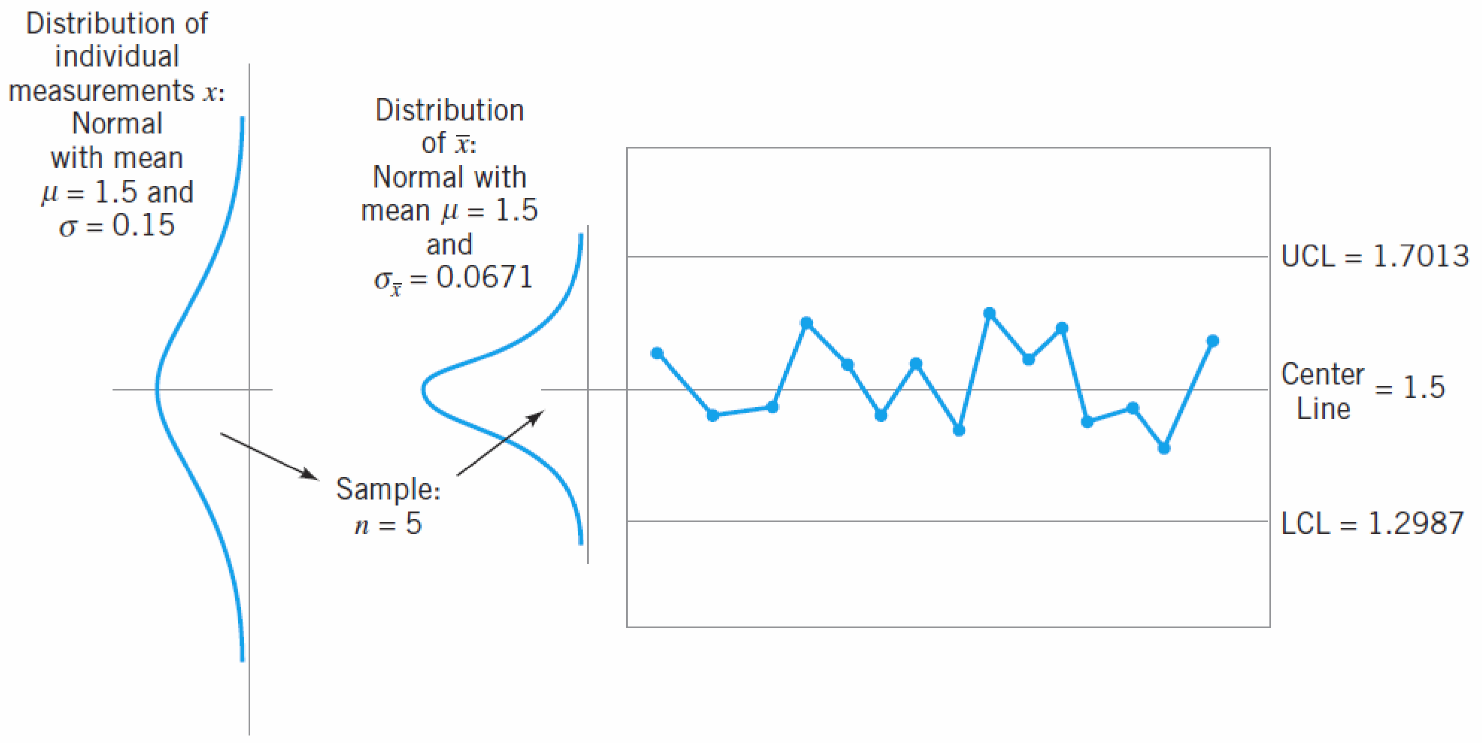


FIGURE 5.4 How the control chart works.

Reasons for Popularity of Control Charts

1. Control charts are a proven technique for improving productivity.
2. Control charts are effective in defect prevention.
3. Control charts prevent unnecessary process adjustment.
4. Control charts provide diagnostic information.
5. Control charts provide information about process capability.

Phase I and Phase II of Control Chart Application

- Phase I is a **retrospective analysis** of process data to construct **trial control limits**
  - Charts are effective at detecting large, sustained shifts in process parameters, outliers, measurement errors, data entry errors, etc.
  - Facilitates identification and removal of assignable causes
- In phase II, the control chart is used to **monitor** the process
  - Process is assumed to be reasonably stable
  - Emphasis is on **process monitoring**, not on bringing an unruly process into control

STATISTICAL QUALITY CONTROL

Control Chart for Variables

Constructing X *bar Control Chart*

- Data:

Sample 1	Sample 2	Sample 3	Sample 4	….	Sample *t*
X11	X21	X31	X41	…	X*t*1
X12	X22	X32	X42	…	X*t*2
.	.	.	.	.	.
X1*n*	X2*n*	X3*n*	X4*n*	…	X*tn*

Phase II Construction --- Everything is assumed known

$$LCL = \mu_{W_i} - 3\sigma_{W_i} = \mu - 3\frac{\sigma}{\sqrt{n}} = \mu - A\cdot\sigma$$
$$CL = \mu_{W_i} = \mu$$
$$UCL = \mu_{W_i} + 3\sigma_{W_i} = \mu + 3\frac{\sigma}{\sqrt{n}} = \mu + A\cdot\sigma$$

Phase I

Estimating Parameters

Μοντέλο ορίων L σίγμα		
UCL	$=$	$\mu_W + L\sigma_W$
$Center\ Line$	$=$	μ_W
LCL	$=$	$\mu_W - L\sigma_W$

- Unbiased estimator of mean

$$\hat{\mu} = \bar{X}.$$

- with $E(\bar{X}) = \mu$ and $SE(\bar{X}) = \dfrac{\sigma}{\sqrt{n}}$.
- S^2 is an unbiased estimator of the population variable $E(S^2) = \sigma^2$.

Phase I

Μοντέλο ορίων L σίγμα		
UCL	=	$\mu_w + L\sigma_w$
$Center\ Line$	=	μ_w
LCL	=	$\mu_w - L\sigma_w$

Estimating Parameters

- On the opposide side S is not an unbiased estimator of σ.
- Ισχύει ότι, $E(S) = c_4(n)\cdot\sigma$ with

$$c_4(n) = \left(\frac{2}{n-1}\right)^{1/2} \frac{\Gamma(n/2))}{\Gamma((n-1)/2)} \approx \frac{4(n-1)}{4n-3}.$$

- Thus, we may estimate σ with

$$\hat{\sigma} = \frac{S}{c_4(n)}.$$

- Moreover,

$$SD(S) = \sigma\sqrt{1 - c_4(n)^2}.$$

Phase I

Estimating Parameters

Μοντέλο ορίων L σίγμα		
UCL	$=$	$\mu_w + L\sigma_w$
$Center\ Line$	$=$	μ_w
LCL	$=$	$\mu_w - L\sigma_w$

- Another estimator is based on R.
- It is known that

$$\hat{\sigma} = \frac{R}{d_2(n)}.$$

with $E(R) = \sigma d_2(n)$.

- Moreover

$$SD(R) = d_3(n) \cdot \sigma$$

Phase I

Estimating Parameters

Μοντέλο ορίων L σίγμα		
UCL	$=$	$\mu_W + L\sigma_W$
$Center\ Line$	$=$	μ_W
LCL	$=$	$\mu_W - L\sigma_W$

- So, we have

$$\hat{\mu} = \overline{\overline{X}} = \frac{\overline{X}_1 + \overline{X}_2 + \Lambda + \overline{X}_m}{m} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij}}{mn}$$

$$E(\overline{\overline{X}}) = \mu \qquad SE(\overline{\overline{X}}) = \frac{\sigma}{\sqrt{n \cdot m}}$$

$$\hat{\sigma} = \frac{R}{d_2(n)}, \hat{\sigma} = \frac{S}{c_4(n)},$$

Φάση I

Constructing X *bar*

- Data:

Sample 1	**Sample 2**	**Sample 3**	**Sample 4**	**….**	**Sample *m***
X11	X21	X31	X41	…	X*m*1
X12	X22	X32	X42	…	X*m*2
.	.	.	.	.	.
X1*n*	X2*n*	X3*n*	X4*n*	…	X*mn*

- Thus, we have

$$LCL = \bar{\bar{X}} - \frac{3 \cdot \bar{S}}{c_4(n) \cdot \sqrt{n}} = \bar{\bar{X}} - A_3(n) \cdot \bar{S}$$
$$CL = \bar{\bar{X}}$$
$$UCL = \bar{\bar{X}} + \frac{3 \cdot \bar{S}}{c_4(n) \cdot \sqrt{n}} = \bar{\bar{X}} + A_3(n) \cdot \bar{S}$$

$$LCL = \bar{\bar{X}} - \frac{3 \cdot \bar{R}}{d_2(n) \cdot \sqrt{n}} = \bar{\bar{X}} - A_2\bar{R}$$
$$CL = \bar{\bar{X}}$$
$$UCL = \bar{\bar{X}} + \frac{3 \cdot \bar{R}}{d_2(n) \cdot \sqrt{n}} = \bar{\bar{X}} + A_2\bar{R}$$

Table for d_2, d_3, c_4...

n	A	A_2	A_3	d_2	d_3	c_4	D_1
2	2.1213	1.88	2.6587	1.1284	0.8525	0.7979	0.
3	1.7321	1.0233	1.9544	1.6926	0.8884	0.8862	0.
4	1.5	0.7286	1.6281	2.0588	0.8798	0.9213	0.
5	1.3416	0.5768	1.4273	2.3259	0.8641	0.94	0.
6	1.2247	0.4832	1.2871	2.5344	0.848	0.9515	0.
7	1.1339	0.4193	1.1819	2.7044	0.8332	0.9594	0.2047
8	1.0607	0.3725	1.0991	2.8472	0.8198	0.965	0.3877
9	1.	0.3367	1.0317	2.97	0.8078	0.9693	0.5465
10	0.9487	0.3083	0.9754	3.0775	0.7971	0.9727	0.6864
11	0.9045	0.2851	0.9274	3.1729	0.7873	0.9754	0.8109
12	0.866	0.2658	0.8859	3.2585	0.7785	0.9776	0.923
13	0.8321	0.2494	0.8495	3.336	0.7704	0.9794	1.0247
14	0.8018	0.2354	0.8173	3.4068	0.763	0.981	1.1177
15	0.7746	0.2231	0.7885	3.4718	0.7562	0.9823	1.2031
16	0.75	0.2123	0.7626	3.532	0.7499	0.9835	1.2823
17	0.7276	0.2028	0.7391	3.5879	0.7441	0.9845	1.3557
18	0.7071	0.1943	0.7176	3.6401	0.7386	0.9854	1.4243
19	0.6882	0.1866	0.6979	3.689	0.7335	0.9862	1.4885
20	0.6708	0.1796	0.6797	3.7349	0.7287	0.9869	1.5489

An example

Sample	1st obs	2nd obs	3rd obs	4th obs	5th obs
1	987,50	987,10	985,00	989,00	987,20
2	987,80	990,10	990,80	989,20	990,50
3	994,60	989,90	988,80	990,20	990,60
4	991,30	985,30	984,50	990,90	989,20
5	988,80	988,80	990,90	990,50	988,70
6	993,10	985,10	987,30	991,90	989,70
7	987,90	991,90	985,60	988,40	991,00
8	990,40	989,80	993,40	991,40	993,70
9	992,80	987,10	991,20	986,00	987,30
10	995,80	991,50	987,10	991,90	988,20
11	991,70	987,50	991,50	987,30	991,60
12	989,20	992,70	990,00	990,10	993,20
13	990,60	992,20	985,80	993,60	986,20
14	995,10	993,60	982,50	987,40	987,10
15	989,40	995,90	987,80	990,70	986,40
16	986,10	991,30	991,50	991,40	985,50
17	993,20	990,20	990,00	986,70	988,20
18	991,50	989,60	988,80	985,30	991,00
19	991,00	990,90	990,00	988,00	995,80
20	985,60	990,60	987,10	988,30	987,10
21	992,60	996,80	994,30	986,20	986,30
22	988,80	995,20	988,00	991,10	986,90
23	990,30	986,40	987,00	985,80	990,20
24	993,80	987,60	989,20	987,90	989,80
25	992,00	992,60	984,30	988,20	988,40
26	993,60	992,70	988,30	988,30	994,60
27	990,50	989,40	995,60	989,90	990,50
28	987,20	989,50	987,50	990,30	987,10
29	986,10	984,80	991,20	982,60	986,70
30	994,00	986,20	990,20	987,50	990,90

m=30 Samples of size n=5

An example

$$\hat{\mu} = \overline{\overline{X}} = \frac{\overline{X}_1 + \overline{X}_2 + \Lambda + \overline{X}_m}{m} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij}}{mn} = 989{,}569$$

$$\overline{S} = \frac{S_1 + S_2 + \Lambda + S_m}{m} = 2{,}715$$

$$\overline{R} = \frac{R_1 + R_2 + \Lambda + R_m}{m} = 6{,}510$$

$$LCL = \overline{\overline{X}} - A_3(n) \cdot \overline{S} = 989{,}569 - 1{,}4273 \cdot 2{,}715 = 985{,}693$$

$$CL = \overline{\overline{X}} = 989{,}569$$

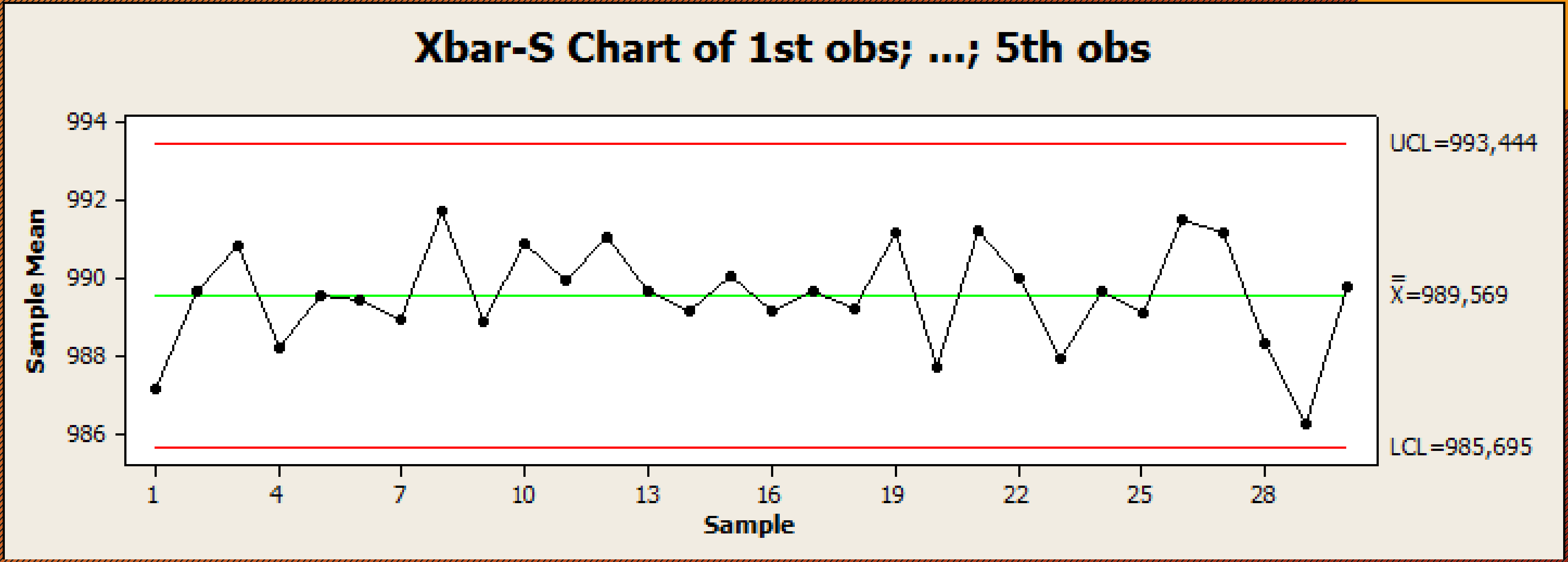
$$UCL = \overline{\overline{X}} + A_3(n) \cdot \overline{S} = 989{,}569 + 1{,}4273 \cdot 2{,}715 = 993{,}444$$

An example

Sample	1st obs	2nd obs	3rd obs	4th obs	5th obs	Sample Mean	Sample SD	Sample Range
1	987,50	987,10	985,00	989,00	987,20	987,160	1,429	4,000
2	987,80	990,10	990,80	989,20	990,50	989,680	1,211	3,000
3	994,60	989,90	988,80	990,20	990,60	990,820	2,216	5,800
4	991,30	985,30	984,50	990,90	989,20	988,240	3,162	6,800
5	988,80	988,80	990,90	990,50	988,70	989,540	1,069	2,200
6	993,10	985,10	987,30	991,90	989,70	989,420	3,276	8,000
7	987,90	991,90	985,60	988,40	991,00	988,960	2,526	6,300
8	990,40	989,80	993,40	991,40	993,70	991,740	1,752	3,900
9	992,80	987,10	991,20	986,00	987,30	988,880	2,946	6,800
10	995,80	991,50	987,10	991,90	988,20	990,900	3,431	8,700
11	991,70	987,50	991,50	987,30	991,60	989,920	2,303	4,400
12	989,20	992,70	990,00	990,10	993,20	991,040	1,787	4,000
13	990,60	992,20	985,80	993,60	986,20	989,680	3,526	7,800
14	995,10	993,60	982,50	987,40	987,10	989,140	5,165	12,600
15	989,40	995,90	987,80	990,70	986,40	990,040	3,656	9,500
16	986,10	991,30	991,50	991,40	985,50	989,160	3,075	6,000
17	993,20	990,20	990,00	986,70	988,20	989,660	2,441	6,500
18	991,50	989,60	988,80	985,30	991,00	989,240	2,452	6,200
19	991,00	990,90	990,00	988,00	995,80	991,140	2,870	7,800
20	985,60	990,60	987,10	988,30	987,10	987,740	1,864	5,000
21	992,60	996,80	994,30	986,20	986,30	991,240	4,794	10,600
22	988,80	995,20	988,00	991,10	986,90	990,000	3,290	8,300
23	990,30	986,40	987,00	985,80	990,20	987,940	2,151	4,500
24	993,80	987,60	989,20	987,90	989,80	989,660	2,486	6,200
25	992,00	992,60	984,30	988,20	988,40	989,100	3,354	8,300
26	993,60	992,70	988,30	988,30	994,60	991,500	2,997	6,300
27	990,50	989,40	995,60	989,90	990,50	991,180	2,513	6,200
28	987,20	989,50	987,50	990,30	987,10	988,320	1,477	3,200
29	986,10	984,80	991,20	982,60	986,70	986,280	3,168	8,600
30	994,00	986,20	990,20	987,50	990,90	989,760	3,052	7,800

Υπολογίζω το δειγματικό μέσο (*sample mean*), τη δειγματική τυπική απόκλιση (*sample SD*) και το δειγματικό εύρος (*sample range*) για κάθε Sample!

An example / Minitab



$$LCL = \bar{\bar{X}} - A_3(n) \cdot \bar{S} = 989{,}569 - 1{,}4273 \cdot 2{,}715 = 985{,}693$$

$$CL = \bar{\bar{X}} = 989{,}569$$

$$UCL = \bar{\bar{X}} + A_3(n) \cdot \bar{S} = 989{,}569 + 1{,}4273 \cdot 2{,}715 = 993{,}444$$

Phase I

Constructing R - Chart

- Data:

Sample 1	**Sample 2**	**Sample 3**	**Sample 4**	**….**	**Sample *m***
X11	X21	X31	X41	…	X*m*1
X12	X22	X32	X42	…	X*m*2
.	.	.	.	.	.
X1n	X2n	X3n	X4n	…	X*mn*

- The limits are

$$LCL = \overline{R} - 3 \cdot d_3(n) \cdot \frac{R}{d_2(n)} = D_3 R$$

$$CL = \overline{R}$$

$$UCL = \overline{R} + 3 \cdot d_3(n) \cdot \frac{R}{d_2(n)} = D_4 R$$

Φάση Ι

Constructing S

- Data

Sample 1	Sample 2	Sample 3	Sample 4	….	Sample m
X11	X21	X31	X41	…	Xm1
X12	X22	X32	X42	…	Xm2
.	.	.	.	.	.
X1n	X2n	X3n	X4n	…	Xmn

- Thus

$$LCL = \bar{S} - 3 \cdot \frac{\bar{S}}{c_4(n)} \cdot \sqrt{1 - c_4(n)^2} = B_3 \bar{S}$$

$$CL = \bar{S}$$

$$UCL = \bar{S} + 3 \cdot \frac{\bar{S}}{c_4(n)} \cdot \sqrt{1 - c_4(n)^2} = B_4 \bar{S}$$

An example

$$\hat{\mu} = \bar{\bar{X}} = \frac{\bar{X}_1 + \bar{X}_2 + \Lambda + \bar{X}_m}{m} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} X_{ij}}{mn} = 989{,}569$$

$$\bar{S} = \frac{S_1 + S_2 + \Lambda + S_m}{m} = 2{,}715$$

$$\bar{R} = \frac{R_1 + R_2 + \Lambda + R_m}{m} = 6{,}510$$

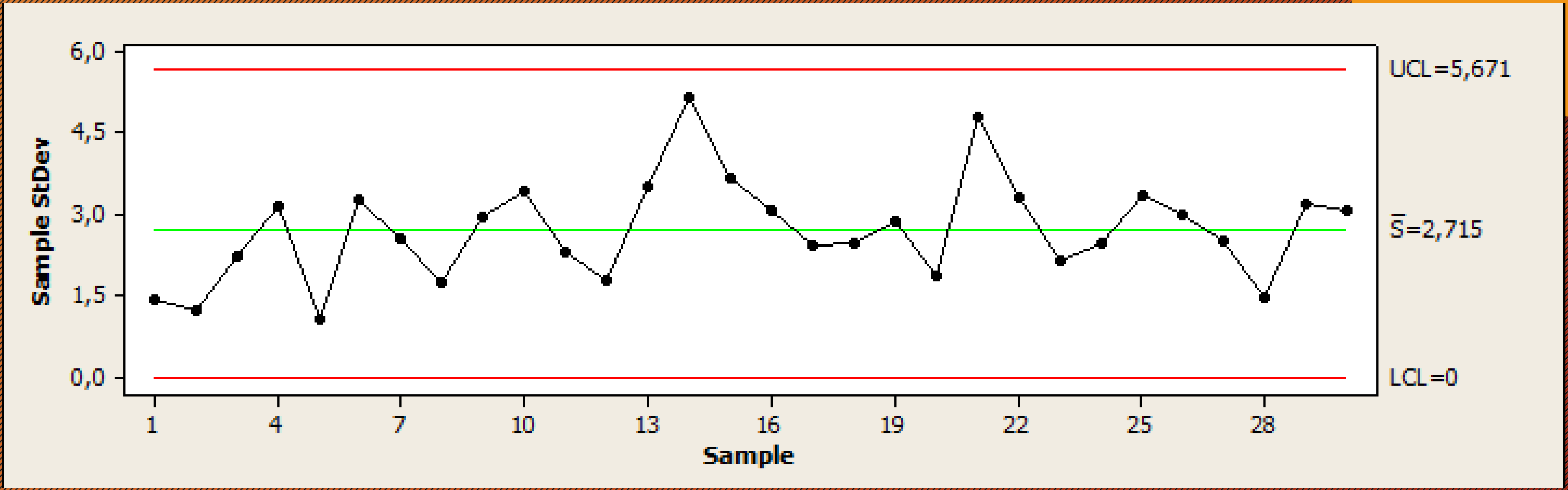
$$LCL = \bar{S} - 3 \cdot \frac{\bar{S}}{c_4(n)} \cdot \sqrt{1 - c_4(n)^2} = 2{,}715 - 3\frac{2{,}715}{0{,}94}\sqrt{1 - 0{,}94^2} = -0{,}241$$

$$CL = \bar{S} = 2{,}715$$

$$UCL = \bar{S} + 3 \cdot \frac{\bar{S}}{c_4(n)} \cdot \sqrt{1 - c_4(n)^2} = 2{,}715 + 3\frac{2{,}715}{0{,}94}\sqrt{1 - 0{,}94^2} = 5{,}671$$

c_4
0.7979
0.8862
0.9213
0.94
0.9515
0.9594
0.965
0.9693
0.9727
0.9754
0.9776
0.9794
0.981
0.9823
0.9835
0.9845
0.9854
0.9862
0.9869

An Example Minitab



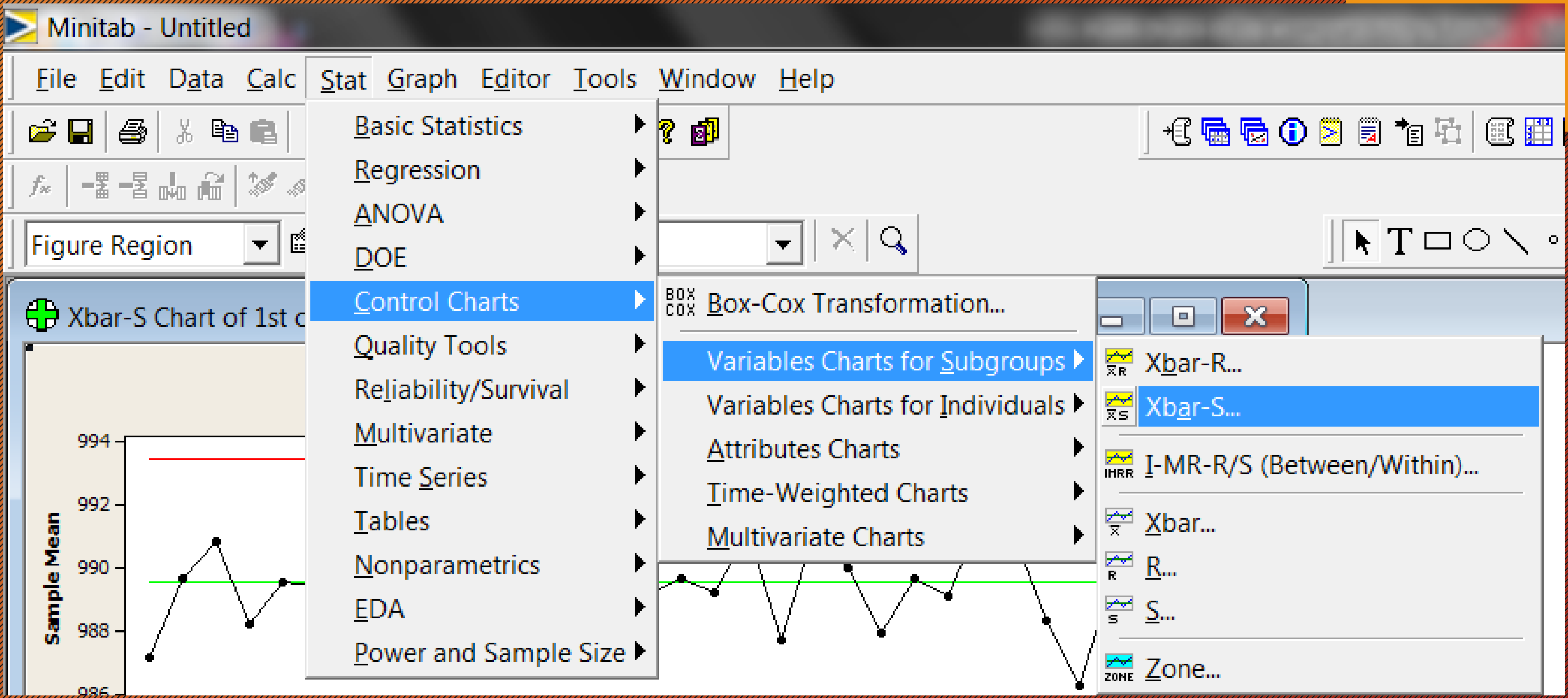
$$LCL = 2{,}715 - 3\frac{2{,}715}{0{,}94}\sqrt{1-0{,}94^2} = -0{,}241$$

?

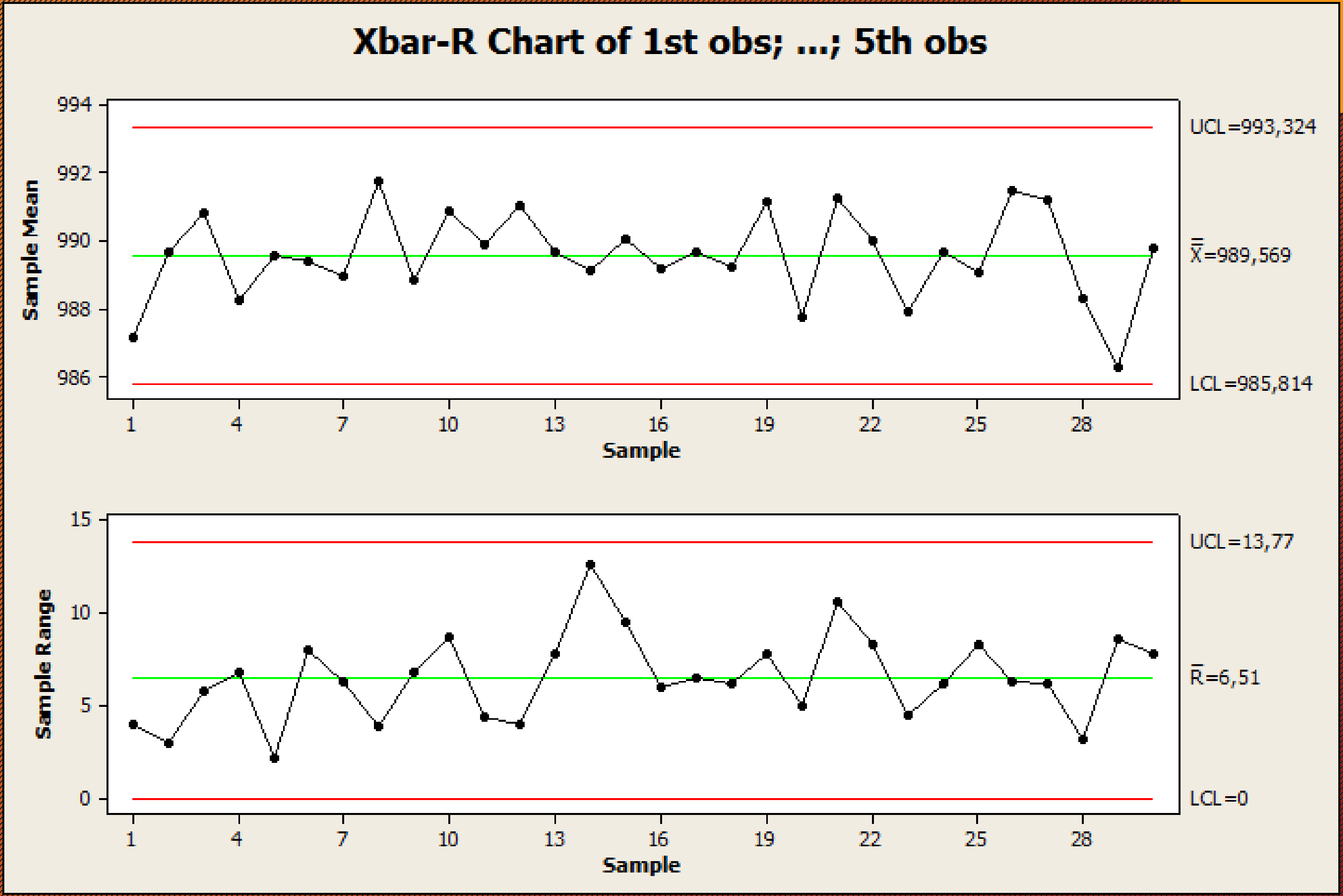
$$CL = 2{,}715$$

$$UCL = 2{,}715 + 3\frac{2{,}715}{0{,}94}\sqrt{1-0{,}94^2} = 5{,}671$$

An Example Minitab



An Example Minitab



Phase II

Constructing R Chart

- Data:

Sample 1	Sample 2	Sample 3	Sample 4	….	Sample t
X11	X21	X31	X41	…	Xt1
X12	X22	X32	X42	…	Xt2
.	.	.	.	.	.
X1n	X2n	X3n	X4n	…	Xtn

- Thus,

$$LCL = d_2(n)\sigma - 3 \cdot d_3(n) \cdot \sigma = D_1 \cdot \sigma$$

$$CL = d_2(n)\sigma$$

$$UCL = d_2(n)\sigma + 3 \cdot d_3(n) \cdot \sigma = D_2 \cdot \sigma$$

Phase II

Constructing S Chart

- Data

Sample 1	Sample 2	Sample 3	Sample 4	….	Sample t
X11	X21	X31	X41	…	Xt1
X12	X22	X32	X42	…	Xt2
.	.	.	.	.	.
X1n	X2n	X3n	X4n	…	Xtn

- Thus,

$$LCL = c_4(n)\cdot\sigma - 3\cdot\sigma\cdot\sqrt{1-c_4(n)^2} = B_5\sigma$$
$$CL = c_4(n)\cdot\sigma$$
$$UCL = c_4(n)\cdot\sigma + 3\cdot\sigma\cdot\sqrt{1-c_4(n)^2} = B_6\sigma$$

STATISTICAL QUALITY CONTROL

Control Charts for individual observations

Control Charts for individual observations

Observation / sample 1	Observation / sample 2	Observation / sample 3	Observation / sample 4	….	Observation / sample t
X1	X2	X3	X4	…	Xt

Phase II

$X_1, X_2, \ldots$ with

$$X_i \sim N(\mu, \sigma^2)$$

Then the control limits will be

$$LCL = \mu_{X_i} - 3\sigma_{X_i} = \mu - 3\sigma,$$
$$CL = \mu_{X_i} = \mu,$$
$$UCL = \mu_{X_i} + 3\sigma_{X_i} = \mu + 3\sigma$$

Obs 1	Obs 2	Obs 3	Obs 4	….	Obs *m*
X1	X2	X3	X4	…	X*t*

Φάση Ι

$X_1, X_2, \ldots, X_m$, with unknown parameters

$$LCL = \hat{\mu}_{X_i} - 3\hat{\sigma}_{X_i} = \overline{X} - 3\frac{\overline{MR}}{d_2},$$

$$CL = \hat{\mu}_{X_i} = \overline{X},$$

$$UCL = \mu_{X_i} + 3\sigma_{X_i} = \overline{X} + 3\frac{\overline{MR}}{d_2}$$

$MR_i = |X_{i+1} - X_i|$, i=1,2,… moving range

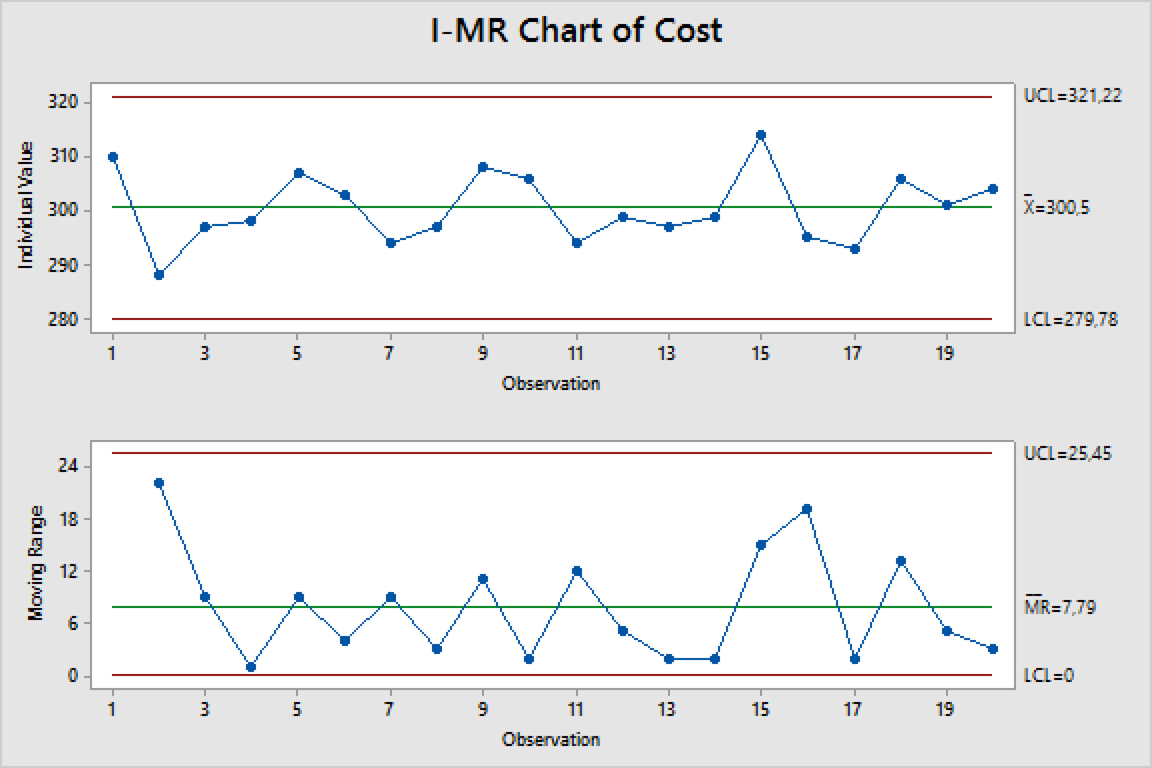
For monitoring variability

$$LCL = D_3\overline{MR} = 0 \cdot \overline{MR},$$

$$CL = \overline{MR},$$

$$UCL = D_4\overline{MR} = 3.267 \cdot \overline{MR}$$

I-MR (MINITAB)



- Data: 310, 288, 297, 298, 307, 303, 294, 297, 308, 306, 294, 299, 297, 299, 314, 295, 293, 306, 301, 304

$$\bar{X} = 300.5, \overline{MR} = 7.79$$

- Στο MINITAB: Stat / Control Charts / Variables Charts for Individuals / I-MR, Variables: Cost, I-MR Options / Estimate / Average moving range

STATISTICAL QUALITY CONTROL

Examples

Example 6.1 The Hard Bake Process

TABLE 6.1

Flow Width Measurements (microns) for the Hard-Bake Process

Sample Number	Wafers 1	2	3	4	5	$\bar{x}_i$	R_i
1	1.3235	1.4128	1.6744	1.4573	1.6914	1.5119	0.3679
2	1.4314	1.3592	1.6075	1.4666	1.6109	1.4951	0.2517
3	1.4284	1.4871	1.4932	1.4324	1.5674	1.4817	0.1390
4	1.5028	1.6352	1.3841	1.2831	1.5507	1.4712	0.3521
5	1.5604	1.2735	1.5265	1.4363	1.6441	1.4882	0.3706
6	1.5955	1.5451	1.3574	1.3281	1.4198	1.4492	0.2674
7	1.6274	1.5064	1.8366	1.4177	1.5144	1.5805	0.4189
8	1.4190	1.4303	1.6637	1.6067	1.5519	1.5343	0.2447
9	1.3884	1.7277	1.5355	1.5176	1.3688	1.5076	0.3589
10	1.4039	1.6697	1.5089	1.4627	1.5220	1.5134	0.2658
11	1.4158	1.7667	1.4278	1.5928	1.4181	1.5242	0.3509
12	1.5821	1.3355	1.5777	1.3908	1.7559	1.5284	0.4204
13	1.2856	1.4106	1.4447	1.6398	1.1928	1.3947	0.4470
14	1.4951	1.4036	1.5893	1.6458	1.4969	1.5261	0.2422
15	1.3589	1.2863	1.5996	1.2497	1.5471	1.4083	0.3499
16	1.5747	1.5301	1.5171	1.1839	1.8662	1.5344	0.6823
17	1.3680	1.7269	1.3957	1.5014	1.4449	1.4874	0.3589
18	1.4163	1.3864	1.3057	1.6210	1.5573	1.4573	0.3153
19	1.5796	1.4185	1.6541	1.5116	1.7247	1.5777	0.3062
20	1.7106	1.4412	1.2361	1.3820	1.7601	1.5060	0.5240
21	1.4371	1.5051	1.3485	1.5670	1.4880	1.4691	0.2185
22	1.4738	1.5936	1.6583	1.4973	1.4720	1.5390	0.1863
23	1.5917	1.4333	1.5551	1.5295	1.6866	1.5592	0.2533
24	1.6399	1.5243	1.5705	1.5563	1.5530	1.5688	0.1156
25	1.5797	1.3663	1.6240	1.3732	1.6887	1.5264	0.3224
						$\Sigma\bar{x}_i = 37.6400$	$\Sigma R_i = 8.1302$
						$\bar{\bar{x}} = 1.5056$	$\bar{R} = 0.32521$

$$\overline{R} = \frac{\sum_{i=1}^{25} R_i}{25} = \frac{8.1302}{25} = 0.32521$$

$$\text{LCL} = \overline{R} D_3 = 0.32521(0) = 0$$

$$\text{UCL} = \overline{R} D_4 = 0.32521(2.114) = 0.68749$$

$$\overline{\overline{x}} = \frac{\sum_{i=1}^{25} \overline{x}_i}{25} = \frac{37.6400}{25} = 1.5056$$

$$\text{UCL} = \overline{\overline{x}} + A_2 \overline{R} = 1.5056 + (0.577)(0.32521) = 1.69325$$

and

$$\text{LCL} = \overline{\overline{x}} - A_2 \overline{R} = 1.5056 - (0.577)(0.32521) = 1.31795$$

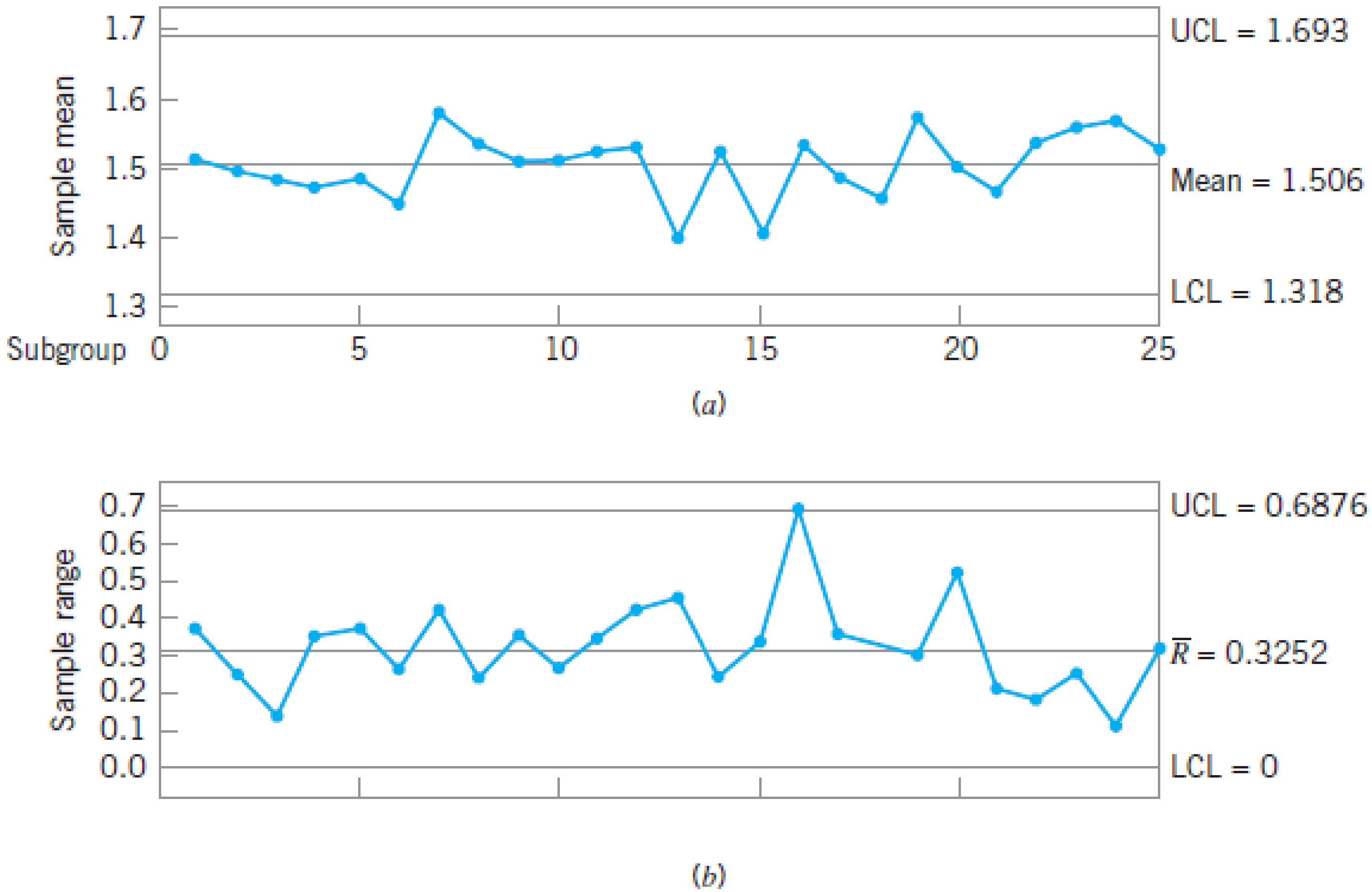


FIGURE 6.2 $\overline{x}$ and R charts (from Minitab) for flow width in the hard-bake process.

■ TABLE 6.2

Additional Samples for Example 6.1

Sample Number	Wafers 1	2	3	4	5	$\bar{x}_i$	R_i
26	1.4483	1.5458	1.4538	1.4303	1.6206	1.4998	0.1903
27	1.5435	1.6899	1.5830	1.3358	1.4187	1.5142	0.3541
28	1.5175	1.3446	1.4723	1.6657	1.6661	1.5332	0.3215
29	1.5454	1.0931	1.4072	1.5039	1.5264	1.4152	0.4523
30	1.4418	1.5059	1.5124	1.4620	1.6263	1.5097	0.1845
31	1.4301	1.2725	1.5945	1.5397	1.5252	1.4724	0.3220
32	1.4981	1.4506	1.6174	1.5837	1.4962	1.5292	0.1668
33	1.3009	1.5060	1.6231	1.5831	1.6454	1.5317	0.3445
34	1.4132	1.4603	1.5808	1.7111	1.7313	1.5793	0.3181
35	1.3817	1.3135	1.4953	1.4894	1.4596	1.4279	0.1818
36	1.5765	1.7014	1.4026	1.2773	1.4541	1.4824	0.4241
37	1.4936	1.4373	1.5139	1.4808	1.5293	1.4910	0.0920
38	1.5729	1.6738	1.5048	1.5651	1.7473	1.6128	0.2425
39	1.8089	1.5513	1.8250	1.4389	1.6558	1.6560	0.3861
40	1.6236	1.5393	1.6738	1.8698	1.5036	1.6420	0.3662
41	1.4120	1.7931	1.7345	1.6391	1.7791	1.6716	0.3811
42	1.7372	1.5663	1.4910	1.7809	1.5504	1.6252	0.2899
43	1.5971	1.7394	1.6832	1.6677	1.7974	1.6970	0.2003
44	1.4295	1.6536	1.9134	1.7272	1.4370	1.6321	0.4839
45	1.6217	1.8220	1.7915	1.6744	1.9404	1.7700	0.3187

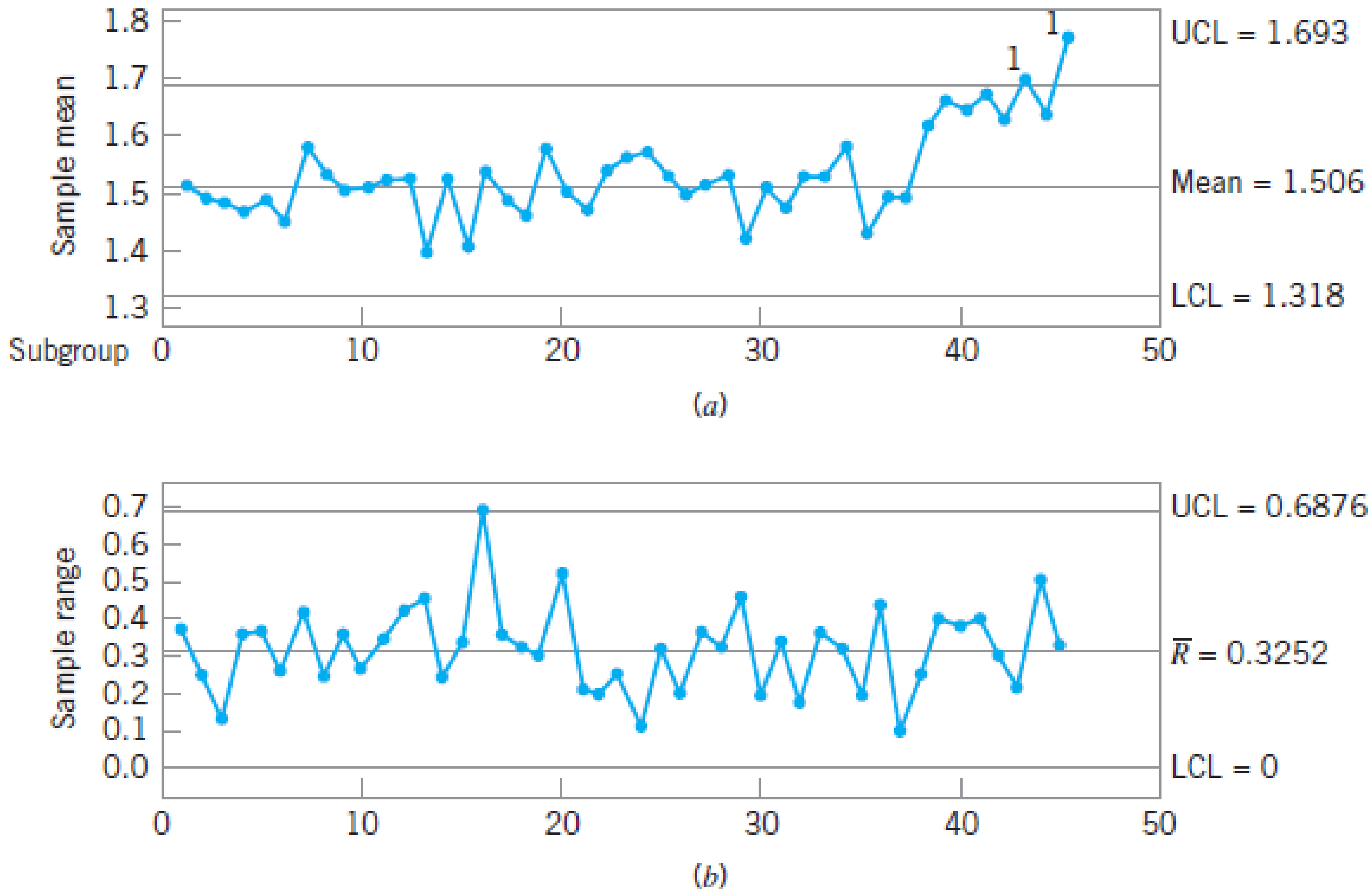


FIGURE 6.4 Continuation of the $\bar{x}$ and R charts in Example 6.1.

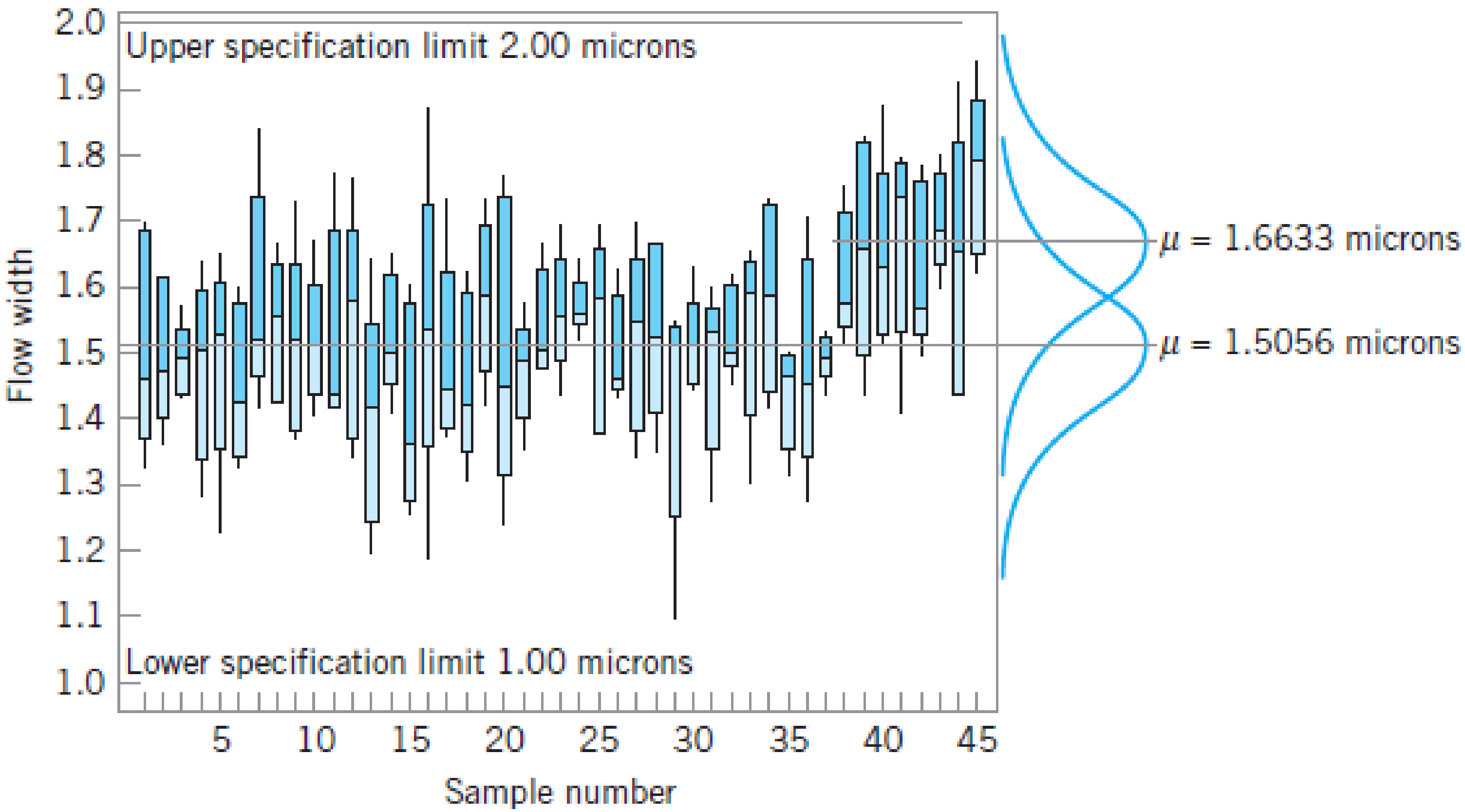


FIGURE 6.5 Tier chart constructed using the Minitab box plot procedure for the flow width data.

Control vs. Specification Limits

- **Control** limits are derived from natural process variability, or the n**atural tolerance** limits of a process
- **Specification** limits are determined externally, for example by customers or designers
- There is no mathematical or statistical relationship between the control limits and the specification limits

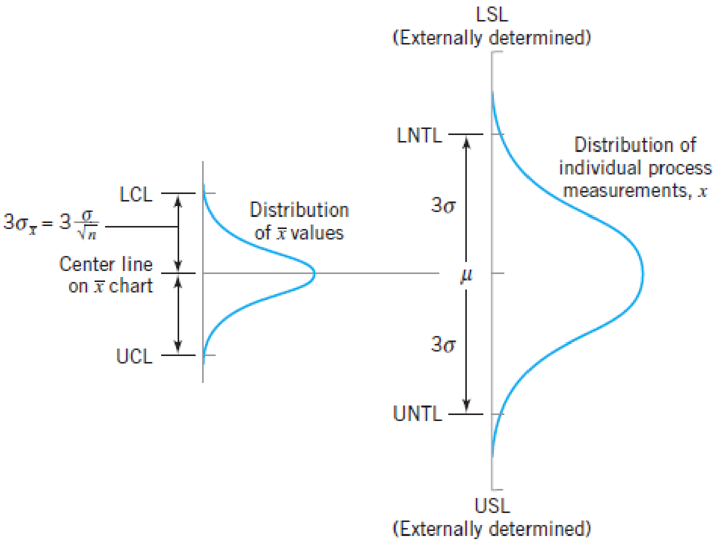
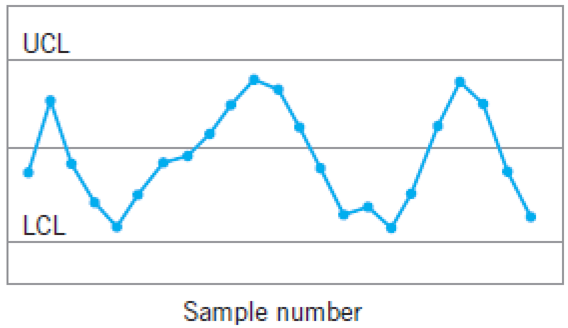
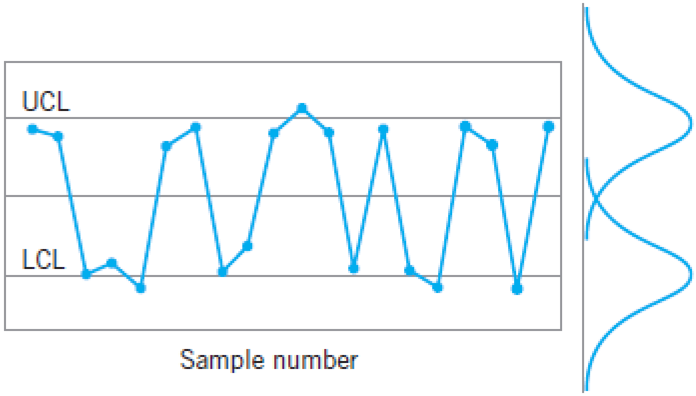


FIGURE 6.6 Relationship of natural tolerance limits, control limits, and specification limits.

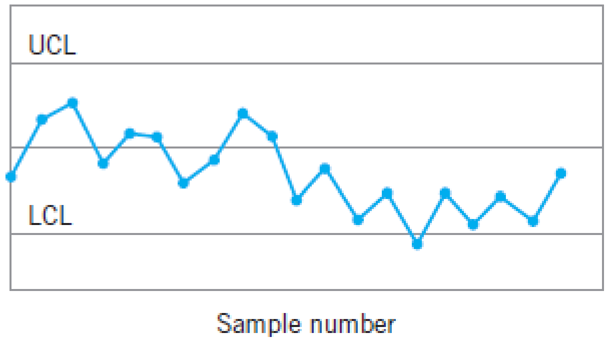
6.2.4 Interpretation of Control Charts



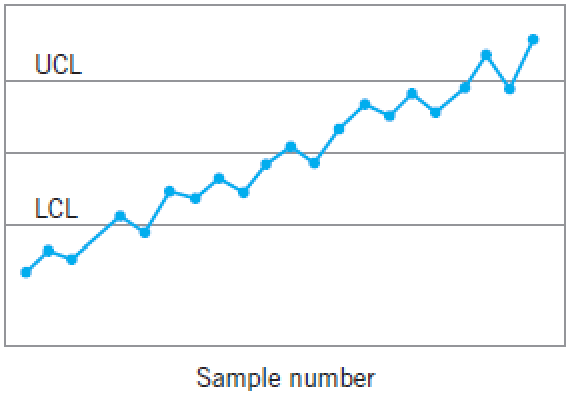
■ FIGURE 6.8 Cycles on a control chart.



■ FIGURE 6.9 A mixture pattern.



■ FIGURE 6.10 A shift in process level.



■ FIGURE 6.11 A trend in process level.

EXAMPLE 6.3 $\bar{x}$ and s Charts for the Piston Ring Data

Construct and interpret $\bar{x}$ and s charts using the piston ring inside diameter measurements in Table 6.3.

SOLUTION

The grand average and the average standard deviation are

$$\bar{\bar{x}} = \frac{1}{25}\sum_{i=1}^{25} \bar{x}_i = \frac{1}{25}(1{,}850.028) = 74.001$$

and

$$\bar{s} = \frac{1}{25}\sum_{i=1}^{25} s_i = \frac{1}{25}(0.2351) = 0.0094$$

respectively. Consequently, the parameters for the $\bar{x}$ chart are

$$\text{UCL} = \bar{\bar{x}} + A_3\bar{s} = 74.001 + (1.427)(0.0094) = 74.014$$

$$\text{CL} = \bar{\bar{x}} = 74.001$$

$$\text{LCL} = \bar{\bar{x}} - A_3\bar{s} = 74.001 - (1.427)(0.0094) = 73.988$$

and for the s chart

$$\text{UCL} = B_4\bar{s} = (2.089)(0.0094) = 0.0196$$

$$\text{CL} = \bar{s} = 0.0094$$

$$\text{LCL} = B_3\bar{s} = (0)(0.0094) = 0$$

The control charts are shown in Figure 6.17. There is no indication that the process is out of control, so those limits could be adopted for phase II monitoring of the process.

EXAMPLE 6.5 Loan Processing Costs

The mortgage loan processing unit of a bank monitors the costs of processing loan applications. The quantity tracked is the average weekly processing costs, obtained by dividing total weekly costs by the number of loans processed during the week. The processing costs for the most recent 20 weeks are shown in Table 6.6. Set up individual and moving range control charts for these data.

TABLE 6.6
Costs of Processing Mortgage Loan Applications

Weeks	Cost x	Moving Range MR
1	310	
2	288	22
3	297	9
4	298	1
5	307	9
6	303	4
7	294	9
8	297	3
9	308	11
10	306	2
11	294	12
12	299	5
13	297	2
14	299	2
15	314	15
16	295	19
17	293	2
18	306	13
19	301	5
20	304	3
	$\bar{x} = 300.5$	$\overline{\text{MR}} = 7.79$

SOLUTION

To set up the control chart for individual observations, note that the sample average cost of the 20 observations is $\bar{x} = 300.5$ and that the average of the moving ranges of two observations is $\overline{\text{MR}} = 7.79$. To set up the moving range chart, we use $D_3 = 0$ and $D_4 = 3.267$ for $n = 2$. Therefore, the moving range chart has center line $\overline{\text{MR}} = 7.79$, LCL = 0, and UCL $= D_4\overline{\text{MR}} = (3.267)7.79 = 25.45$. The control chart (from Minitab) is shown in Figure 6.19*b*. Notice that no points are out of control.

For the control chart for individual measurements, the parameters are

$$\text{UCL} = \bar{x} + 3\frac{\overline{\text{MR}}}{d_2}$$
$$\text{Center line} = \bar{x} \qquad (6.33)$$
$$\text{LCL} = \bar{x} - 3\frac{\overline{\text{MR}}}{d_2}$$

If a moving range of $n = 2$ observations is used, then $d_2 = 1.128$. For the data in Table 6.6, we have

$$UCL = \bar{x} + 3\frac{\overline{MR}}{d_2} = 300.5 + 3\frac{7.79}{1.128} = 321.22$$

$$\text{Center line} = \bar{x} = 34.088$$

$$LCL = \bar{x} - 3\frac{\overline{MR}}{d_2} = 300.5 - 3\frac{7.79}{1.128} = 279.78$$

The control chart for individual cost values is shown in Figure 6.19*a*. There are no out-of-control observations on the individuals control chart.

The interpretation of the individuals control chart is very similar to the interpretation of the ordinary $\bar{x}$ control chart. A shift in the process mean will result in a single point or a series of points that plot outside the control limits on the control chart for individuals. Sometimes a point will plot outside the control limits on both the individuals chart and the moving range chart. This will often occur because a large value of x will also lead to a large value of the moving range for that sample. This is very typical behavior for the individuals and moving range control charts. It is most likely an indication that the mean is out of control and not an indication that both the mean and the variance of the process are out of control.

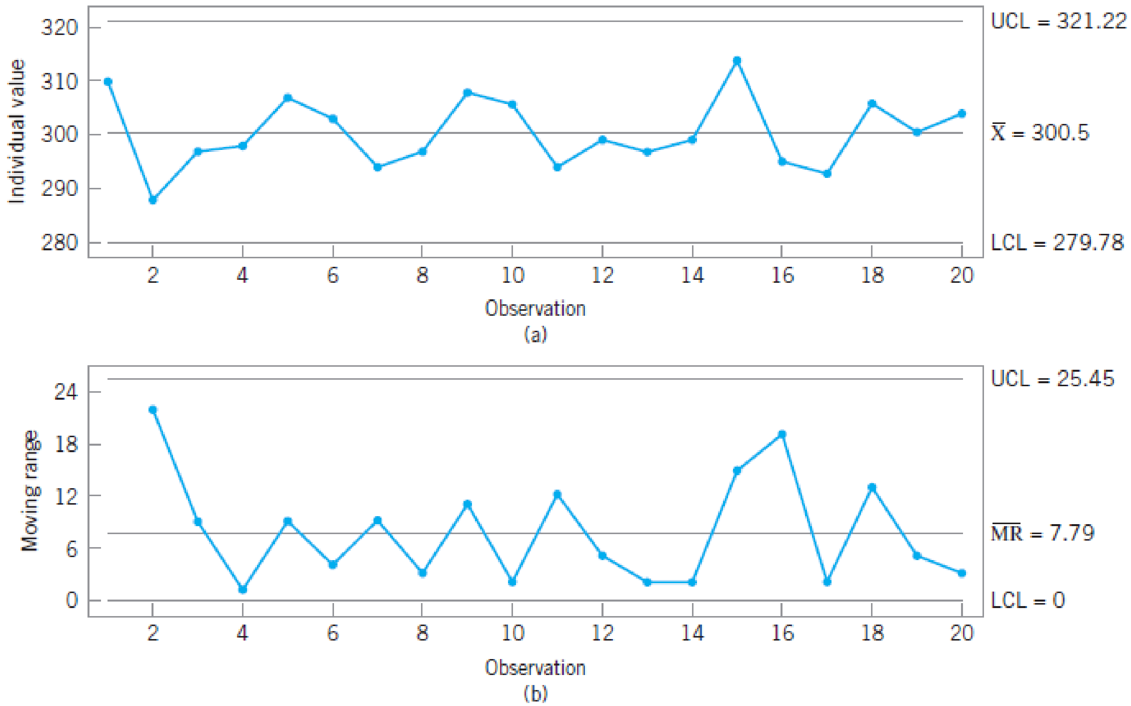


FIGURE 6.19 Control charts for (*a*) individual observations on cost and for (*b*) the moving range.

■ TABLE 6.7

Costs of Processing Mortgage Loan Applications, Weeks 21–40

Week	Cost x	Week	Cost x
21	305	31	310
22	282	32	292
23	305	33	305
24	296	34	299
25	314	35	304
26	295	36	310
27	287	37	304
28	301	38	305
29	298	39	333
30	311	40	328

FIGURE 6.20 Continuation of the control chart for individuals and the moving range using the additional data in Table 6.7.

STATISTICAL QUALITY CONTROL

Control Charts for Attributes

Control Charts for Attributes

- ☑ For variables that are categorical
  - ☑ Good/bad, yes/no, acceptable/unacceptable
- ☑ Measurement is typically counting defectives
- ☑ Charts may measure
  - ☑ Percent defective (p-chart)
  - ☑ Number of defects (c-chart)

Control Limits for p-Charts

Population will be a binomial distribution, but applying the Central Limit Theorem allows us to assume a normal distribution for the sample statistics

$$UCL_p = \bar{p} + z\sigma_{\hat{p}}$$

$$LCL_p = \bar{p} - z\sigma_{\hat{p}}$$

$$\sigma_{\hat{p}} = \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$$

where
- $\bar{p}$ = mean fraction defective in the sample
- z = number of standard deviations
- $\sigma_{\hat{p}}$ = standard deviation of the sampling distribution
- n = sample size

p-Chart for Data Entry

Sample Number	Number of Errors	Fraction Defective	Sample Number	Number of Errors	Fraction Defective
1	6	.06	11	6	.06
2	5	.05	12	1	.01
3	0	.00	13	8	.08
4	1	.01	14	7	.07
5	4	.04	15	5	.05
6	2	.02	16	4	.04
7	5	.05	17	11	.11
8	3	.03	18	3	.03
9	3	.03	19	0	.00
10	2	.02	20	4	.04

Total = 80

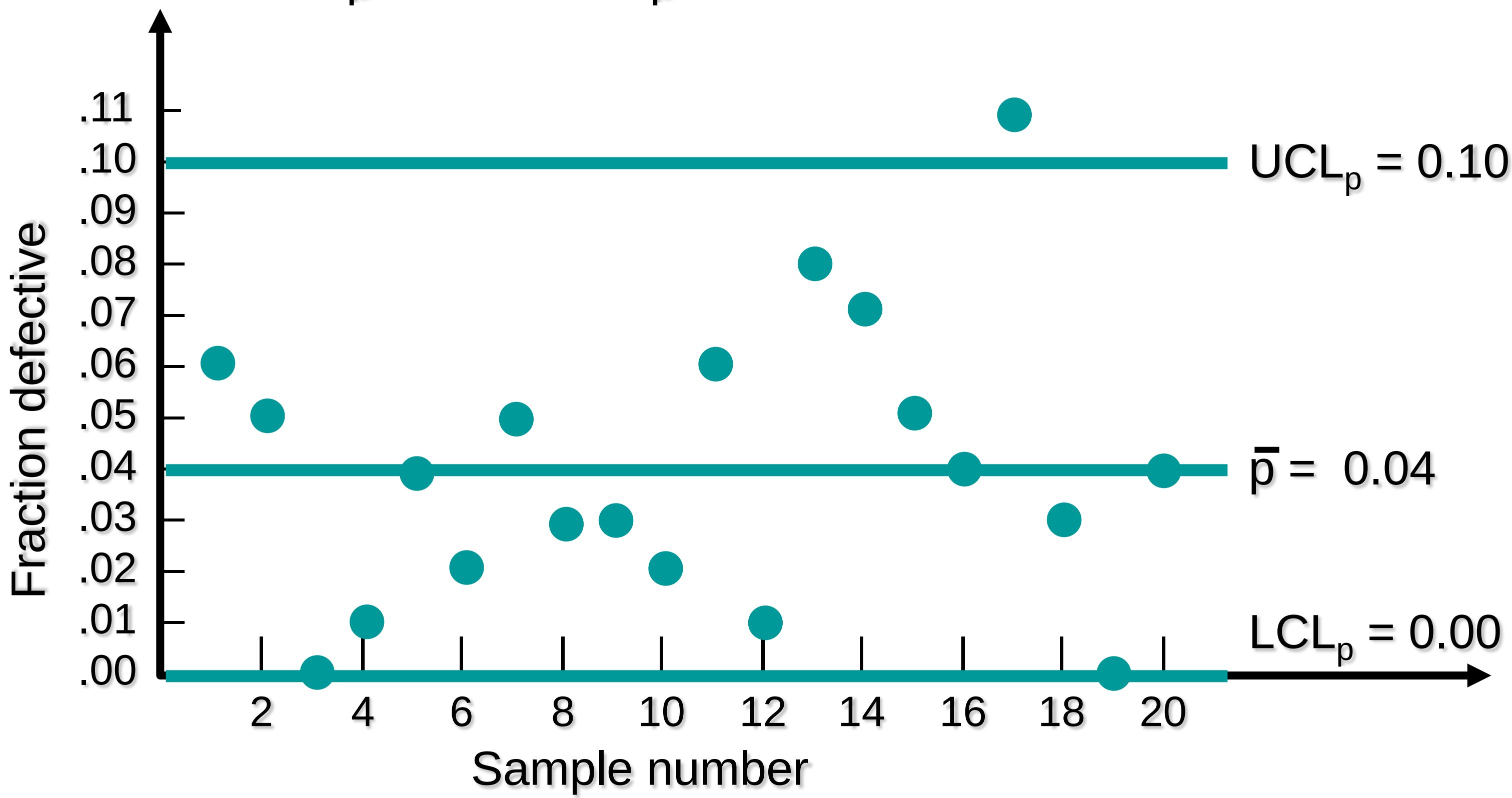
$$\bar{p} = \frac{80}{(100)(20)} = .04$$

$$\sigma_{\hat{p}} = \sqrt{\frac{(.04)(1 - .04)}{100}} = .02$$

p-Chart for Data Entry

$$UCL_p = \bar{p} + z\sigma_{\hat{p}} = .04 + 3(.02) = .10$$

$$LCL_p = \bar{p} - z\sigma_{\hat{p}} = .04 - 3(.02) = 0$$

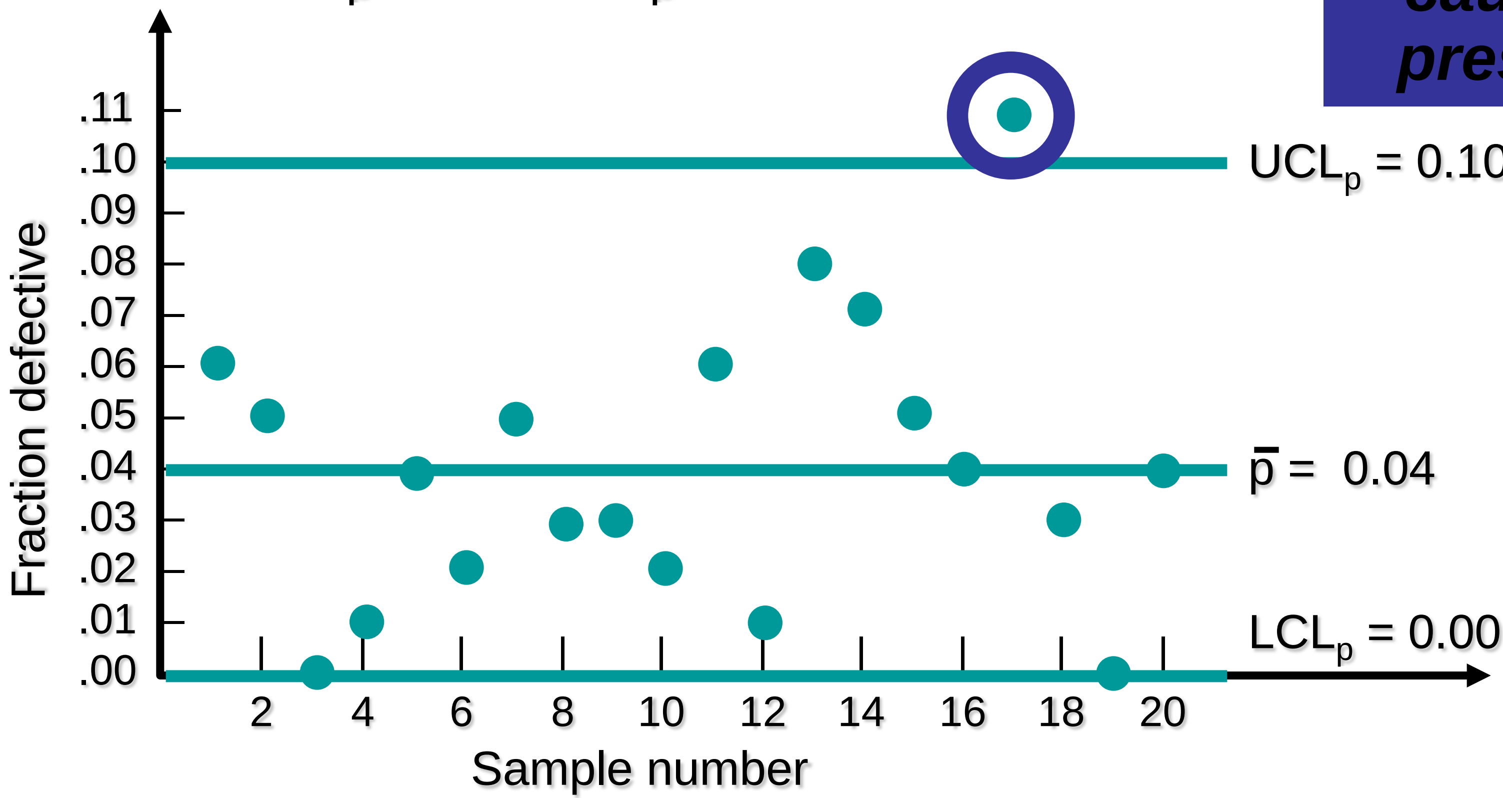


p-Chart for Data Entry

$$UCL_p = \bar{p} + z\sigma_{\hat{p}} = .04 + 3(.02) = .10$$

$$LCL_p = \bar{p} - z\sigma_{\hat{p}} = .04 - 3(.02) = 0$$

Possible assignable causes present



Control Limits for c-Charts

Population will be a Poisson distribution, but applying the Central Limit Theorem allows us to assume a normal distribution for the sample statistics

$$UCL_c = \bar{c} + 3\sqrt{\bar{c}} \qquad LCL_c = \bar{c} - 3\sqrt{\bar{c}}$$

where $\bar{c}$ = mean number defective in the sample

c-Chart for Cab Company

$\bar{c}$ = 54 complaints/9 days = 6 complaints/day

$$UCL_c = \bar{c} + 3\sqrt{\bar{c}} = 6 + 3\sqrt{6} = 13.35$$

$$LCL_c = \bar{c} - 3\sqrt{\bar{c}} = 6 - 3\sqrt{6} = 0$$